Accuracy of Pricing Models for CAT Bonds
– An Empirical Analysis

by Marcello Galeotti*, Marc Gürtler**, and Christine Rehan***

Abstract. (CAT-astrophe Bonds are of significant importance in the field of alternative risk transfer. Since the market of CAT Bonds is not as liquid as e.g. the stock market, the use of pricing models is of high relevance. One important parameter in all pricing models is the probability of catastrophe. Consequently, there are two possibilities of determination. On the one hand, if we have an expectation regarding the probability of catastrophe, the value of the bond can be calculated using a model. On the other hand, if the price of the bond is known, we can implicitly calculate the probability of catastrophe as anticipated by market participants. This probability is used in this paper in order to determine a method to measure accuracy of pricing models. Furthermore, an adjustment for cyclic, seasonal and business cyclic effects is established with the objective of verifying their effects on pricing of CAT Bonds. Having identified the most accurate CAT Bond Pricing Model, we use it in order to find further pricing determining factors.

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1 Introduction and Motivation

Since both the trend of insured losses and the trend of numbers of catastrophes is positive, (re-)insurance companies have to consider new ways of coping with the risk. One possibility is to transfer the risk from the reinsurance market to financial markets. Important Financial Instruments which are used therefore are CAT Bonds. The volume of CAT Bond principal outstanding rose to USD 13.8 billion in 2007 and further growth is expected. The main idea of a CAT Bond is that a (re-)insurer issues the bond to protect against high losses due to a specified catastrophe. An investor buys the bond in order to diversify one’s portfolio and to receive high yields resulting from the covered risk.

Pricing models for CAT Bonds are rare in the literature. However, one can interpret a CAT Bond as a portfolio consisting of a variable interest bond and an option. In this case, pricing models for CAT Options can be considered to evaluate CAT Bonds. One of the first important models was developed by Cummins/Geman (1995) and addresses the pricing of both catastrophe Insurance Futures and Call Spreads. They suggested an arbitrage-based model by using a Poisson jump process to represent the event of a catastrophe. For the stochastic timing of claims they proposed a geometric Brownian motion. Dassios/Jang (2003) proposed a doubly stochastic Poisson process which allows incorporating reporting lags of the occurred claims. Lee/Yu (2002) suggested an arbitrage approach which takes into account moral hazard and basis risk. Typically, the arbitrage pricing methodology does not yield a closed form solution and thus should be handled with Monte Carlo Simulation. Albrecher/Hartinger/Tichy (2003) developed a Quasi Monte Carlo Method which improved the results.

The main challenge while pricing CAT Instruments with these arbitrage-based models is the presence of catastrophe risk. Usually a portfolio which replicates this payment structure cannot be found. Thus, the market for CAT Instruments is incomplete and no unique equivalent martingale measure exists. Hence, a unique price cannot be derived. Apart from that, the modelling of the catastrophic claim process is unapparent.

Against the background of these problems, models have been developed which do not use the assumption of a complete market. These models are based on preferences. Preference-based

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1 See Münchener Rück (2007), p. 47.
3 For a more precise description of the functionality of CAT Bonds see for instance Guy Carpenter (2006) and Gürtler/Rehan (2008).
4 See Berge (2005), pp. 163.
approaches use a utility framework for maximizing the agent profits. A problem is the specification of the utility functions and the assumption of uniform preferences of all agents. For instance preference-based approaches are proposed by AASE (1999) and EMBRECHTS/MEISTER (1995).

Models directly considering CAT Bonds can be separated into three approaches, which are the binomial pricing, the indifference pricing and the premium calculation model. The binomial pricing model was first mentioned in the literature by TILLEY (1997) and CANABARRO ET AL (1998). NGUYEN (2007) examined the binomial pricing model for the one period and for the multi period case. Also TILLEY (1997) developed multi-period binomial models with different assumptions. A theoretical foundation of this approach has been developed by COX/PEDERSEN (2000) without using the assumption of market completeness. Instead another critical assumption is used. The cash flows of the CAT Bond are considered to depend only on catastrophe risk variables. This is not necessarily true.

Beyond binomial pricing models there exist indifference pricing models. An advantage of the latter is that they remain valid for incomplete markets. YOUNG (2004) proposed such an approach for CAT Bonds. EGAMI/YOUNG (2008) modified it in order to consider structured CAT Bonds. The overall problem which arises when using one of the just mentioned models is the structure of the CAT Bond market. Since the market for CAT Bonds lacks transparency and every contract is individually designed, it is almost impossible to verify the use of the pricing models by empirical analysis.

The third types of models directly considering CAT Bonds are the premium calculation models which are somehow different from the other mentioned pricing models. While the other pricing models take the premiums which are paid by the (re-)insurers for granted, these approaches define as “price” of a CAT Bond the determination of the premium which has to be set. In this approach the price which is also referred to as spread consists of the expected value of loss plus a load for risk margin and expenses. For a theoretical modelling of spreads, LANE (2000) proposed a simple model using empirical data. Based on his experience LANE (2008A) and LANE/MAHUL (2008) proposed yet another model in order to allow for cycle adjustments. MAJOR/KREPS (2003) used a log-linear relationship in order to describe pricing of spreads. Apart from that, WANG (2000) developed a probability transformation which can be used to explain the spreads.

In the following the whole catastrophe securitization shall be called CAT Bond transaction. The CAT Bond is issued by the Special Purpose Vehicle (SPV) in order to guarantee the
sponsor a catastrophe covering up to the limit. The payout of a CAT Bond due to a specified catastrophe is defined by trigger mechanisms. Basically, there are the options of using the height of actual losses of the sponsor (indemnity trigger), a physical measure like the Richter-scale (parametric trigger) or a specified index (index trigger) as trigger mechanisms. All of them are susceptible to basis risk and moral hazard to a certain extent.\textsuperscript{6}

The transaction between the SPV and the sponsor is called alternative reinsurance. The binomial and the indifference pricing model can therefore be classified as pricing models for CAT Bonds, whereas the premium calculation models can be allocated in the alternative reinsurance part. Figure 1 provides an overview of the different pricing approaches for CAT Bond transactions.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Basic structure of a CAT Bond transaction}
\end{figure}

However, the main focus of this paper lies in the comparison of some selected pricing models in the field of premium calculation models.\textsuperscript{7} To our knowledge literature lacks such a comparison of different pricing models. Therefore, Chapter 2 provides fundamentals of premium calculation models, before the selected models are described more in detail. These models are used in chapter 3 in order to carry out an empirical analysis. This analysis is established on the basis of CAT Bond prices within the period 1999 – 2008. We suggest a methodology in order to determine accuracy on the basis of the probability of catastrophe. Furthermore, we examine whether an adjustment of cyclic, seasonal and business cyclic effects improves the results, since these effects seem to have a significant impact on the pricing of CAT Bonds, as observed, for example, by LANE (2008A). Furthermore, the linear model is used for the examination of effects of CAT Bond specific factors on the pricing mechanism.

\textsuperscript{6} See GUY CARPENTER (2007), pp. 27 for a detailed description of trigger mechanisms for CAT Bonds.

\textsuperscript{7} The comparison of, for example binomial models and indifference pricing models, would demand the specified structure of a CAT Bond transaction. This data is not available to the public.
2 Premium calculation models

2.1 Fundamentals

Premium calculation models apply to the alternative reinsurance part of a CAT Bond transaction. In this case the premiums paid by the reinsurer in order to receive protection against losses are determined. When dividing the premiums by the insured limit one receives the number Rate on Line (ROL). The ROL is then paid to the investors by the SPV and determines the price of the CAT Bond. In literature the ROL, which is also called spread, is often referred to as the price of a CAT Bond. This price is paid to investors in addition to the risk free rate. Figure 2 provides an overview of the above described relations.

\[
X_{(a,a+h)} = \begin{cases} 
0, & \text{if } X \leq a \\
X - a, & \text{if } a < X \leq a + h \\
h, & \text{if } a + h < X 
\end{cases}
\]

The question arises how to determine the spread. In order to answer such a question, we start with explaining the usual reinsurance pricing.

Usually, in (re-)insurance pricing the risk is divided into several layers. Assume we have a non-negative random loss variable \(X\). Then a layer with attachment point (point of first loss) \(a\) and limit (point of last loss) \(h\) is defined as

\[ \text{ROL} = \text{Premiums/Limit} = \text{Spread} = \text{Price} \]

\[ r = \text{risk free interest rate} \]

\text{Figure 2: Overview Premium calculation models}

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9 An overview of this kind of pricing is provided by WANG (2003).

10 See for an example FROOT (2001), pp.542.

This means that if the loss is less than or equal to the attachment point, then there occurs no loss to the layer \((a,a+h]\). If the loss \(X\) lies between the attachment point and the limit, then the layer loss is given by \(X - a\). If the loss exceeds the limit, then the loss charged to the layer is the limit.

The loss variable \(X\) has a cumulative distribution function \(F_X(x) = P(X \leq x)\). Accordingly, \(S_X(x) = 1 - F_X(x) = P(X > x)\) denotes the so-called loss exceedance probability curve. This is an important index, since we are interested in the loss which is occurring above a determined level. Assume that there also exists the density function \(f_X(x)\) and, thus, \(s_X(x) = S'_X(x)\).

The question arises of how the reinsurance company is able to determine the loss exceedance probability curve. Usually, geophysical commercial models are used in order to evaluate the curve and to determine the consequent exposure of their own portfolio. The relation between the risk variable \(X\) and the decumulative distribution function can be described as follows.

\[
S_{X,(a,a+h)}(y) = \begin{cases} 
S_X(a + y) = P[X > a + y], & \text{if } 0 \leq y < h \\
0, & \text{if } h \leq y
\end{cases}
\]

This means that the distribution function generates the probability of loss in the interval \((a,a+h]\). Beyond this interval the probability distribution equals 0.

Since \(S_{X,(a,a+h)}(y)\) and also \(s_X(x)\) are continuous on the interval \((a,a+h]\), the expected value is given by:

\[
E[X_{(a,a+h)}] = \int_0^\infty y \cdot s_{X,(a,a+h)}(y)dy = \int_a^{a+h} x \cdot s_X(x)dx
\]

Using partial integration one receives the following simplified formula, which is referred to in the literature as the net premium:

\[
E[X_{(a,a+h)}] = \int_a^{a+h} S_X(x)dx
\]

In general and without any layer assumptions, the net premium can be written as:

\[\text{(3)} \quad E[X_{(a,a+h)}] = \int_a^{a+h} S_X(x)dx\]

\[\text{(4)} \quad E[X_{(a,a+h)}] = \int_a^{a+h} S_X(x)dx\]

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12 The main geophysical commercial models are provided by AIR, RMS and EQECAT. A detail description of these models can be found in NGUYEN (2007), pp. 287 and STRASSBURGER (2006), pp. 31. See STRASSBURGER (2006), pp. 75 for a description and discussion of the different types of loss exceedance curves.


\begin{equation}
(5) \quad E[X] = \int_0^\infty S_x(x) \, dx
\end{equation}

Since the net premium is a lower bound for a premium, a risk load is demanded. In literature there are various possibilities in order to account for a risk load.\textsuperscript{15} The problem is that many of these principles do not fulfil the requirements of a coherent risk measure which have been introduced by ARTZNER ET AL. (1999). WANG (2000) proposed the class of distortion functions which fulfil the above stated requirements and can be regarded as an appropriate risk measure.

Through examination of the loss exceedance curve $S_x(x)$, one can follow a slightly different approach by determining three numbers of the layer $(a, a+h]$ which are the probability of first loss (PFL) with $PFL = S_x(a) = P(X > a)$, the conditional expected loss (CEL) with $CEL = E[X \mid a < X \leq a + h]$ and the probability of last loss (PLL) with $PLL = S_x(a+h) = P(X > a + h)$.

Using these numbers, the expected loss (EL) can be derived by the following formula:

\begin{equation}
(6) \quad EL = P[X \leq a] \cdot E[X \mid X \leq a] + P[X > a] \cdot \frac{E[X \mid a < X \leq a + h]}{CEL}
\end{equation}

\begin{equation}
(7) \quad \Rightarrow EL = PFL \cdot CEL
\end{equation}

The $EL$ of a layer is also referred to as the spread or price of a layer in case of risk neutrality. If the market is complete then the risk neutral price of a layer is the $EL$ of this layer.\textsuperscript{16} Since, usually, there is no market completeness, a risk premium is demanded.\textsuperscript{17} An important question is how the $EL$ can be transformed to include a risk load and describe the market price of the risk.

Empirically it can be stated that the observed market price $EL'$ for layer $X_{(a,a+h]}$ always includes a risk load $\lambda$ in addition to the $EL$.\textsuperscript{18}

\begin{equation}
(8) \quad EL' = ROL = EL + \lambda
\end{equation}

The coupon payment to investors then equals

\textsuperscript{15} See EMBRECHTS (1996) and ZWEIFEL/EISEN (2003), pp. 243 for an overview of premium calculation principles.

\textsuperscript{16} See FROOT (2001), pp. 537 for a discussion of problems while using the expected loss as a pricing approach.

\textsuperscript{17} See chapter 1.

\textsuperscript{18} See WANG (2004) and LANE (2000), p. 269. According to LANE (2000) the price for the security consists of $EL$ and $\lambda$, whereas $\lambda$ can be regarded as the net price.
Let us assume that the expected probability of first loss \( PFL' \) already includes the determined probability of first loss \( PFL \) plus a risk load \( \lambda \).

\[
(9) \quad c = r + \frac{EL + \lambda}{EL}
\]

This implies that \( \lambda \) includes \( PFL' \), which has to be adjusted by means of the determined probability \( PFL \).

\[
(10) \quad PFL' = PFL + \lambda
\]

The expected probability of catastrophe can therefore be determined by

\[
(11) \quad EL' = \frac{PFL \cdot CEL}{EL} + (PFL' - PFL)CEL = PFL \cdot CEL + PFL' \cdot CEL - PFL \cdot CEL
\]

When this approach is considered, the question arises how the relationship between observed market prices and the evaluated \( EL \), and thereby the one between the implied probability of catastrophe and the examined probability, can be modelled.

According to BANTWAL/KUNREUTER (1999) there arises a CAT Bond premium puzzle. They found that the spreads are too high to be explained only by investor risk aversion. They suggested considering also ambiguity aversion, myopic loss aversion and fixed costs of education as impact factors for the high spreads. They further suggested standardizing the structures of a CAT Bond transaction in order to allow for fair prices.

BERGE (2005) accomplished a multivariate regression analysis in order to determine factors explaining the risk loads of CAT Bonds additionally to the \( EL \). Mainly he found that risks which arise out of market imperfections have an impact on the level of the risk load. MAJOR/KREPS (2003) found that the relationship between the spread and the determined \( EL \) is best explained via a log-linear function. LANE (2000) first used a Cobb-Douglas Production function and later developed another approach in LANE (2008A) to allow for consideration of cyclic effects. This approach was followed by the analysis of LANE/MAHUL (2008) where a linear relationship was assumed.

In the following, we will examine the proposed pricing models for CAT Bonds which are used for the empirical analysis, more precisely the models by LANE (2000/2008A), MAJOR/KREPS (2003) and WANG (2000/2004).
2.2 Lane’s approach

LANE (2000) proposed a pricing model inspired by empirical observations. He stated that $\lambda$ can be represented by a Cobb-Douglas production function of the probability of first loss ($PFL$) and the conditional expected loss ($CEL$):\(^{19}\)

$$\lambda = \gamma (PFL)^\alpha (CEL)^\beta$$

- $\gamma, \alpha, \beta$: constants set by fitting the equation to empirical data.

After observing the market over some years, LANE (2008A) concluded that the approach using Cobb-Douglas production functions was not appropriate. Instead he suggested allowing for cyclic adjustments in the linear model.\(^{20}\) Therefore, he proposed the use of a cyclic Index $CI_i$ which is developed in LANE (2007) and LANE/MAHUL (2008).\(^{21}\) He proposed a simple linear model to fit the spreads and included a factor for cyclic effects:

$$EL_i = CI_i \cdot [a + b \cdot EL]$$
$$\Rightarrow EL_i / CI_i = a + b \cdot EL$$

Recently LANE/MAHUL (2008) proposed yet another method for the inclusion of cyclic effects. They suggested the use of a multiple linear regression:

$$EL_i = a \cdot EL + b \cdot CI_i + \lambda$$

Their results will be discussed in the context of the empirical analysis in chapter 3.

2.3 Major/Kreps approach

MAJOR/KREPS (2003) established an empirical analysis to identify influencing factors for the price of a CAT Bond. Therefore, they used the following linear model

$$\ln(EL_i) = a + b \cdot \ln(EL) + c \cdot F_1 + \ldots + k \cdot F_k$$

$a,\ldots,k$ Constants
$F_1,\ldots,F_k$ Factors such as geographic location or lead reinsurer

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\(^{19}\) See LANE (2000), p. 271. For detailed information about the Cobb-Douglas Production Function see COBB/DOUGLAS (1928).

\(^{20}\) Examples of selected empirical data suggest that cyclic effects are an important factor for the price development. See LANE (2008A), p. 3.

\(^{21}\) See LANE/MAHUL (2008), Annex 2 for the data of the index.
\(\ln(EL)\) had the biggest impact on \(\ln(EL')\), as expected. In their paper they do not exactly describe why they consider a logarithmic relationship. They just used a logarithmic scale in order to plot the determined \(EL\) against the \(ROL\) and found the relationship:

\[
0,47(EL^{0.53}) = ROL
\]

This leads directly to the relationship \(0,47 \cdot 0,53 \cdot \ln(EL) = \ln(ROL)\). This observation yields to their examination as presented in (16). MAJOR/KREPS (2003) did not compare their approach to other approaches which could be the linear relationship.

### 2.4 Wang's probability transformation

In chapter 2.1, the net premium of a loss variable has been derived. It has been stated that usually a risk load is demanded. WANG (1996/2000) proposed a class of distortion operators in order to allow for a risk load. He considered the transformation \(S'(x) = g[S(x)]\), where \(g : [0,1] \rightarrow [0,1]\) is increasing with \(g(0) = 0\) and \(g(1) = 1\). Thus, he proposed a premium calculation model by

\[
H(X) = \int_0^\infty g[S_X(x)]dx
\]

where \(S_X(x)\) denotes the decumulative distribution function of the loss variable \(X\) as defined above. Furthermore, the function \(g : [0,1] \rightarrow [0,1]\) needs to fulfil four necessary criteria in order to allow for a coherent risk measure.

The four necessary criteria are such that \(g\) should be increasing \((g'(u) \geq 0)\), to ensure that a probability distribution is defined. Moreover, \(g\) should be concave \((g''(u) \leq 0)\), to ensure that the risk load is non-negative and that, for a fixed limit, the relative risk loading increases as the attachment point increases. For the definitions of valid probabilities after applying the distortion operator, it is necessary that \(g(0) = 0, g(1) = 0, 0 < g(u) < 1, 0 < u < 1\). Finally, \(g'(0) = +\infty\) is demanded in order to ensure unbounded relative loading at extremely high layers.\(^{22}\)

After verifying these criteria for the distortion operator, one can summarize the desirable properties. WANG (1996/2000) specified some of them. Basically, they are consistent with the

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properties of a coherent risk measure as defined by ARTZNER ET AL (1999), namely translation invariance, subadditivity, positive homogeneity and monotonicity.

Assuming this theoretical background, Wang proposed the following distortion operator:

$$g_\lambda(u) = \Phi[\Phi^{-1}[u] + \lambda], \Phi$$ being the standard Gauss cdf, with $\lambda > 0$.

Posing $u = S_\lambda(x)$ and $g_\lambda(u) = S_{\lambda}^*(x)$ one gets the probability transformation

$$S_{\lambda}^*(x) = \Phi[\Phi^{-1}[S_\lambda(x)] + \lambda]$$

which transforms the probability of attaching the layer into an empirical probability of attaching the layer including a risk load.

The above stated necessary criteria for a premium calculation principle as required by WANG (1996/2000) can be verified easily. Moreover, the desirable properties for a coherent risk measure can be shown.23

3 Empirical Analysis

3.1 Motivation and data

In the literature, the pricing models as described above have been developed and analysed. To our knowledge the literature lacks a comparison of the accuracy of different pricing models for the case of CAT Bonds which is our objective in this analysis. For this purpose, we suggest how accuracy of the models can be measured and identify the most appropriate model. Briefly summarized, we carry out different linear regressions in order to explain the relationship between the determined $\text{EL}$ and the observed $\text{EL}^*$. We will concentrate on this relationship, since all studies identified the determined expected loss as the most significant variable when explaining the price of a CAT Bond. Since our main objective lies in the comparison of the different proposed models, we neglect other influencing factors in the first step.

In the second step, the pricing models are adjusted for cyclic and seasonal effect. We verify the assumption stated by LANE (2008A) that cyclic effects are of significant importance for pricing CAT Bonds. For this purpose, we adjust the pricing data by a cyclic index and examine if the model fitting increases after the adjustment.

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23 See WANG (2000), pp. 20. The desirable properties for receiving a coherent risk measure are shown by using the notation of the Choquet integral. The representation was introduced and established primarily by CHOQUET (1953/54). DENNEBERG (1992) gave a sound description of the theory. For the Choquet integral the main properties of a coherent risk measure have been shown in DENNEBERG (1994), pp. 71.
The empirical analysis uses data sets provided by Lane Financial LLC in which 224 CAT Bond transactions between the years 1999 and mid 2008 are specified. The data include, in particular, values of the above mentioned $PFL$, $PLL$, $CEL$ and, respectively, the $EL$. Furthermore, the spread to LIBOR$^{24}$ is given which can be used to generate the implied probability of catastrophe $PFL^*$. Therefore, we assume the $CEL$ to be constant.$^{25}$ Using this assumption, the $PFL^*$ can be simply determined by equation (12).

3.2 Accuracy of the proposed models

We suggest comparing the different premium calculation models by finding the model with the best explanation for the variation of a regression. For the application of the Wang transformation the probabilities of layer attachment are necessary, so we will carry out every analysis by using the parameters $PFL$ and $PFL^*$, as derived through the data.

The first model is the one proposed by LANE (2008A) which uses the method of a simple linear regression in order to adjust the $EL$ and $EL^*$. Instead we use the numbers $PFL$ and $PFL^*$, respectively, which can be trivially determined by

$$PFL^* = a \cdot PFL + \lambda.$$  

The second model follows the suggestion by MAJOR/KREPS (2003) of describing the relationship between $EL$ and $EL^*$ as a log-linear one. Again using the numbers $PFL$ and $PFL^*$, one obtains

$$\ln(PFL^*) = a \cdot \ln(PFL) + \lambda.$$  

The third model is based on the Wang transformation as proposed by WANG (2000/2004). Using $PFL = S_X(x) = P(X > x)$ and $PFL^* = S_X^*(x) = P(X > x)$ one can derive from (20):

$$PFL^* = \Phi(\Phi^{-1}(PFL) + \lambda).$$  

In order to apply the linear regression method, we use a generalisation of the Wang transformation with a factor $a$. In dependence on WANG (2000) the generalisation results in the trans-

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$^{24}$ The spread premium to LIBOR is consistent with the above mentioned $EL^* = EL + \lambda$. Further, we use the adjusted spread premium to LIBOR which means that the value is converted from a 360-day year measure to a 365-day year to be consistent with numbers as, for instance, PFL and PLL (see e.g. LANE (2007), p. 20).

$^{25}$ The assumption of a constant $CEL$ is appropriate since it can be stated that market participants have estimations about the $PFL$. 

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formation \( g_a(u) = \Phi[a \cdot \Phi^{-1}(u) + \lambda] \) with \( g : [0,1] \rightarrow [0,1] \) increasing, \( g(0) = 0 \) and \( g(1) = 1 \) and \( a > 0 \).\(^{26}\)

Using the generalised transformation, one obtains the following equation as a model description of the linear regression:\(^{27}\)

\[
\Phi^{-1}(PFL^*) = a \cdot \Phi^{-1}(PFL) + \lambda \quad \forall a, \lambda \geq 0.
\]

Performing a linear regression analysis with the three models, one receives the values \( R^2 \) denoting the fraction of variance explained by the model.\(^{28}\) Table 1 provides an overview of the results of the regression analysis. The linear model yields the highest \( R^2 \) followed by the generalised Wang Transformation and the log-Regression.

<table>
<thead>
<tr>
<th>Predictor variables</th>
<th>Standard error of the estimator</th>
<th>adjusted ( R^2 )</th>
<th>( R^2 )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(constant), ( PFL )</td>
<td>0.0276</td>
<td>0.766</td>
<td>0.767</td>
<td>0.876</td>
</tr>
<tr>
<td>(constant), ( \ln(PFL) )</td>
<td>0.366</td>
<td>0.659</td>
<td>0.661</td>
<td>0.813</td>
</tr>
<tr>
<td>(constant), ( \Phi^{-1}(PFL) )</td>
<td>0.188</td>
<td>0.693</td>
<td>0.694</td>
<td>0.833</td>
</tr>
</tbody>
</table>

Table 1: Results of the regression analysis

However, in order to come to an appropriate conclusion, the model significance and model assumptions need to be tested. Therefore, in Table 2 the resulting coefficients are presented. In the case of a simple linear regression, the non-standardised coefficients can be used to explain the parameter estimators, which result for the linear model as:

\[
PFL^* = 0.041 + 2.31 \cdot PFL.
\]

In the present case of a simple linear regression, it is adequate to consider the \( t \) Test of the constant and the independent variable in order to decide whether the model is significant or not and thus, whether the model variables are significantly different from zero. In literature it is assumed, that only coefficients with values of \( |t| > 2 \) are significant ones.\(^{29}\) Considering the results of the \( t \) Test in Table 2, it can be observed that only the linear model and the log-

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\(^{26}\) Evidence of this transformation has been given by GAO/QIU (2002). All properties of the original transformation remain valid. The Wang transformation results with the factor \( a = 1 \).

\(^{27}\) See Appendix A.1.

\(^{28}\) An \( R^2 \) of, for example 0.767, means that about 76 % of the data can be explained via the regression function. See GREENE (2003), pp. 31 for more information on the parameter \( R^2 \).

\(^{29}\) See GREENE (2003), pp. 50 and PODDIG ET AL (2001), pp. 279.
Regression provide coefficients which are significantly different from zero. Thus, the results of the generalised Wang transformation can not be considered to be significant.

<table>
<thead>
<tr>
<th>Model</th>
<th>non standardised coefficients</th>
<th>standardised coefficients</th>
<th>T</th>
<th>significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>standard error</td>
<td>Beta</td>
<td></td>
</tr>
<tr>
<td>Linear Model</td>
<td>(constant)</td>
<td>0.041</td>
<td>0.003</td>
<td>15.176</td>
</tr>
<tr>
<td>dep. variable: PFL</td>
<td></td>
<td>2.31</td>
<td>0.085</td>
<td>27.058</td>
</tr>
<tr>
<td>log- Regression</td>
<td>(constant)</td>
<td>-0.677</td>
<td>0.094</td>
<td>-7.237</td>
</tr>
<tr>
<td>dep. variable: ln(PFL)</td>
<td></td>
<td>0.44</td>
<td>0.021</td>
<td>20.792</td>
</tr>
<tr>
<td>Gen. Wang Transformation</td>
<td>(constant)</td>
<td>0.009</td>
<td>0.064</td>
<td>0.137</td>
</tr>
<tr>
<td>dep. variable: ( \Phi^{-1}(PFL) )</td>
<td></td>
<td>0.649</td>
<td>0.029</td>
<td>22.434</td>
</tr>
</tbody>
</table>

Table 2: Coefficients of the regression analysis

The regression analysis leads to the conclusion that the linear model is the most accurate model according to our method and without an adjustment for seasonal and cyclic effects. The Wang Transformation has also been tested as a Pricing Framework for Longevity-Linked Securities by BAUER ET AL (2008). They found that the Wang Transformation does not lead to appropriate prices due to the specific structure of longevity derivatives. They also refer to PELSSER (2008) who states that prices resulting from a Wang Transformation are not consistent with arbitrage-free pricing. He concludes that the Wang Transformation cannot be used as a “universal framework for pricing financial and insurance risks”. Since the general arbitrage-free pricing is not applicable in the case of CAT Bonds it remains an unresolved question whether the Wang Transformation is an appropriate pricing framework in this special case.

However, our analysis found that existing prices can be explained better by using the linear model than the Wang Transformation. Assuming that the linear model is the most appropriate model for explaining spreads and thus for explaining the implicit probability of catastrophe, one can modify the analysis in order to find more factors explaining the spreads. BERGE (2005) followed this idea and accomplished a multiple linear regression analysis for explaining spreads as already mentioned in chapter 2.1. Another analysis using the linear model has

30 See Appendix A.2 for the test of model assumptions.
31 It remains to future studies to point out whether some of the arguments also apply for the case of CAT Bonds.
been established by LANE / MAHUL (2008). They used the linear model in order to adjust the EL to the observed spreads of 247 CAT Bonds. Their results are quite similar to ours.32

3.3 Adjustment of cyclic effects

It is generally accepted in the field of (re-)insurance research that the traditional (re-)insurance market is affected by insurance cycles. MAGUHN (2007) provides an overview concerning different definitions for insurance cycles. Generally, it can be stated that after a soft market which can be identified by relatively low prices and new market participants the market turns into a hard market with relatively high prices. In the special case of catastrophe reinsurance, FROOT (2001) argues that cyclic effects triggered by catastrophe events can be observed.33

In the literature, on the one hand it is stated that the CAT Bond market is less affected by insurance cyclic effects than the reinsurance market.34 On the other hand LANE (2007/2008A) assumes that cyclic effects have a main impact on the pricing mechanisms of CAT Bonds.35 As already mentioned in chapter 2.2 he suggested a cyclic index which is based on catastrophe reinsurance prices. The main problem regarding this index is the fact that the index has not been developed by using statistical methods. Instead, price changes from one year to another have been used. A problem in finding an appropriate index is the unavailability of appropriate data sets. For insurance cycles the cycle lasts, for example, about 7,4 years for Germany and about 7,9 years for Italy.36 Assuming that a CAT Bond cycle index would also last 7 years approximately, we do not have enough CAT Bond data to establish a time series analysis.37 Thus, we will use the cyclic index as proposed by LANE (2007) in order to verify if the adjustment for cyclic effects is improving or changing the results of the comparison of the models.

The first adjustment of cyclic effects has been carried out by utilizing a multiple linear regression:38

\[ PFL^* = a \cdot PFL + b \cdot CI_t + \lambda \]

33 See FROOT (2001), pp. 128.
34 See BELONSKY ET. AL. (1999), p. 160. They argue that the CAT Bond market is more compelling for (re)insurers when the insurance market is a hard one.
35 See LANE (2008), p. 3 for an example of how the status of the cycle has an impact on pricing.
36 See MAGUHN (2007), pp. 53 for more information.
37 Time series analysis is an accurate method in order to verify insurance cycles. See MAGUHN (2007), pp. 49. Since the data basis of CAT Bonds is limited to approximately 10 years by now, a time series analysis could not identify the cyclic effects.
38 This approach was also followed by LANE/MAHUL (2008).
For the log-Regression and the generalised Wang transformation the numbers $PFL'$ and $PFL$ are replaced by $\ln(PFL')$, $\ln(PFL)$ and $\Phi^{-1}(PFL')$, $\Phi^{-1}(PFL)$, respectively.

The results of the analysis are shown in Table 3. No improvement of the model results could be determined. Apart from that, the test of significance of the variables did not improve. Instead, the included cyclic variable should not be included in the model according to the $t$ test. As an alternative approach, the data of the cyclic index has been logarithmized and squared but the modifications did not result in any model improvements.

LANE /MAHUL (2008) established a similar analysis with the main difference that they only considered the cyclic adjustment of the linear model. They also did not receive better results of their regression analysis by the use of this approach.40

<table>
<thead>
<tr>
<th>Predictor variables</th>
<th>$R$</th>
<th>$R^2$</th>
<th>adjusted $R^2$</th>
<th>Standard error of the estimator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Model</td>
<td>0.876</td>
<td>0.768</td>
<td>0.766</td>
<td>0.028</td>
</tr>
<tr>
<td>log-regression</td>
<td>0.815</td>
<td>0.664</td>
<td>0.660</td>
<td>0.366</td>
</tr>
<tr>
<td>Gen. Wang Transformation</td>
<td>0.834</td>
<td>0.696</td>
<td>0.694</td>
<td>0.188</td>
</tr>
</tbody>
</table>

Table 3: Regression results with Cyclic Index

We also considered another approach as proposed by LANE/MAHUL (2008).41 Using the probabilities of catastrophe and a constant $CEL$, Equation (14) results in

\[
PFL^* = CI_i \cdot [a + b \cdot PFL]
\]

\[
\Rightarrow PFL^* / CI_i = a + b \cdot PFL
\]

Again, for the log-regression and the generalised Wang Transformation the numbers $PFL^*$ and $PFL$ have been replaced as described above.

The results of the regression analysis using this approach worsened. To be more precise, the $R^2$ goes down, the variable significance does not improve and the residuals are autocorrelated.42

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39 See Appendix A.3 I.
40 See LANE/MAHUL (2008), pp. 10.
41 See chapter 2.2.
42 See Appendix A.3 II.
The question remains how these results can be interpreted. Either cyclic effects do just not have any impact on the pricing of CAT Bonds or the development of the index is not appropriate. In any case a further development of the cyclic index could provide more clarity.

3.4 Adjustment of seasonal and business cyclic effects

Apart from cyclic effects also seasonal effects can be observed when examining CAT Bonds. LANE (2007) states that before and after wind seasons prices rise and fall due to the expectation of higher losses in this season. We verify if seasonal effects have any impact on the proposed pricing models. Therefore, the index \( SI \) as proposed by LANE (2008B) is used in order to adjust our data for seasonal effects.\(^{43}\)

LANE did not carry out a theoretically based approach in order to identify seasonal effects but he averaged the monthly price shifts for years where no loss occurred. For the analysis we used the index as another predictor for the regression analysis like it was done for the cyclic adjustment in chapter 3.3. For this purpose the index was standardised to the amount of 1 and is called \( SI_S \). The values are shown in Table 4.

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mrz</th>
<th>Apr</th>
<th>Mai</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Okt</th>
<th>Nov</th>
<th>Dez</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI_S</td>
<td>1.003</td>
<td>1.0007</td>
<td>0.9988</td>
<td>0.9982</td>
<td>0.9969</td>
<td>0.9982</td>
<td>1.0001</td>
<td>1.0019</td>
<td>1.0041</td>
<td>1.0028</td>
<td>1.000</td>
<td>0.9996</td>
</tr>
</tbody>
</table>

Table 4: Seasonal index standardised

The model which can be used according to this method provides

\[
(28) \quad PFL_{adj}^* = a \cdot PFL + b \cdot SI_S + \lambda
\]

Using the seasonal index as a dependent variable, one only receives results for the regression analysis which are not significant regarding the \( t \) value as it is shown in Table 5. Also the measure \( R^2 \) did not improve.\(^{44}\)

At this stage we have to conclude that by using the above stated seasonal index, no significant effects can be discovered. The explanation of this result seems quite obvious. The seasonal Index only shows very slight changes as can be observed in Table 4. Thus, the seasonal effects are not distinctive enough as to affect the pricing mechanism of CAT Bonds significantly.

\(^{43}\) See LANE (2008B), p. 5.

\(^{44}\) See Appendix A.4.
### Table 5: Regression Analysis with Seasonal Index

<table>
<thead>
<tr>
<th>Model</th>
<th>dep. variable</th>
<th>Model</th>
<th>dep. variable</th>
<th>coeff. (B)</th>
<th>standard error</th>
<th>Beta</th>
<th>T</th>
<th>significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Model</td>
<td>PFL</td>
<td>non standardised coefficients</td>
<td>constant</td>
<td>0.366</td>
<td>1.079</td>
<td>0.339</td>
<td>0.735</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>PFL</td>
<td>2.307</td>
<td>0.086</td>
<td>0.876</td>
<td>26.880</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SI</td>
<td>-0.325</td>
<td>1.080</td>
<td>-0.010</td>
<td>-0.301</td>
<td>0.763</td>
</tr>
<tr>
<td>log- Regression</td>
<td>ln(PFL)</td>
<td>standardised coefficients</td>
<td>constant</td>
<td>-1.398</td>
<td>14.260</td>
<td>-0.098</td>
<td>0.922</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SI</td>
<td></td>
<td>ln(PFL)</td>
<td>0.440</td>
<td>0.021</td>
<td>0.814</td>
<td>20.758</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>SI</td>
<td></td>
<td>SI</td>
<td>0.726</td>
<td>14.272</td>
<td>0.002</td>
<td>0.051</td>
<td>0.960</td>
</tr>
<tr>
<td>Gen. Wang Transformation</td>
<td>PFL</td>
<td></td>
<td>constant</td>
<td>-0.805</td>
<td>7.323</td>
<td>-0.110</td>
<td>0.913</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Φ⁻¹(PFL)</td>
<td></td>
<td>Φ⁻¹(PFL)</td>
<td>0.648</td>
<td>0.029</td>
<td>0.834</td>
<td>22.389</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>SI</td>
<td></td>
<td>SI</td>
<td>0.815</td>
<td>7.329</td>
<td>0.004</td>
<td>0.111</td>
<td>0.912</td>
</tr>
</tbody>
</table>

Similar results were received by examining business cyclic effects. The GDP quarterly percent changes for the US were included in a multiple regression analysis equivalent to the adjustment of seasonal effects. The results of the regression analysis lead to the conclusion that CAT Bond prices are not suspected by business cyclic effects. This finding supports the assumption of the independence of capital markets and CAT Bond markets, which is widely assumed in the literature.

#### 3.5 Pricing determining factors

The linear model has been identified as the most accurate one according to our method without any changes when considering cyclic, seasonal or business cyclic effects. We use this insight in order to identify pricing determining factors for CAT Bonds. The knowledge of such factors allows for a better understanding of pricing dynamics. MAJOR/KREPS (2003) provides an analysis of determining factors using 46 sets of CAT Bonds transaction data. We extend this analysis by considering 165 examined cases for the period 1999-2007 and include further possible pricing determining factors.

Apart from the determined expected loss the following analysis includes some important CAT Bond specific factors like the type of trigger mechanism, the Rating of the Bond according to

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45 Data have been provided by www.ecomagic.com.
46 See Appendix A.5 for the analysis.
47 Obviously, this is not a proof of the statement but an indication for independence. For the assumption of independence we refer to LITZENBERGER ET AL. (1996) and a recent paper by LANE (2009).
48 In contrast to the study of MAJOR/KREPS (2003), we did not include factors as, for instance, coinsurance and the lead reinsurer as these factors turned out not to be statistically significant anyway.
Standard & Poors, the insured risk, the newness of the asset class and the maturity. Data provided by Standard & Poors are used for this analysis. The following model is examined using a stepwise linear regression method:

\[
E'L = a + b \cdot EL + \left[ c_1 \cdot \left[ \begin{array}{c} \text{Trigger}_\text{Indemnity} \\ \text{Trigger}_\text{Index} \\ \text{Trigger}_\text{Parametric} \end{array} \right] + d_1 \cdot \left[ \begin{array}{c} \text{Risk}_\text{Earthquake} \\ \text{Risk}_\text{Hurricane} \\ \text{Risk}_\text{HurricaneEarthquake} \\ \text{Risk}_\text{Windstorm} \end{array} \right] + e_1 \cdot \left[ \begin{array}{c} \text{Rating}_\text{A} \\ \text{Rating}_\text{B} \\ \text{Rating}_\text{BB} \end{array} \right] + f_1 \cdot \left[ \begin{array}{c} \text{Maturity}_\text{12m} \\ \text{Maturity}_\text{13-24m} \\ \text{Maturity}_\text{25-36m} \\ \text{Maturity}_\text{37-48m} \end{array} \right] + g_1 \cdot \left[ \begin{array}{c} \text{Issuedate}_\text{2000} \\ \text{Issuedate}_\text{2001-2002} \\ \text{Issuedate}_\text{2003-2004} \\ \text{Issuedate}_\text{2005-2006} \end{array} \right] + \epsilon \right]
\]

\[E'L\] observed Expected Loss (annual) (see (8))
\[EL\] determined Expected Loss (annual) (see (7))
\[Trigger\] binary variables, separated due to different mechanisms
\[Risk\] binary variables, separated due to insured risk
\[Rating\] binary variables, separated due to S&P Rating
\[Maturity\] binary variables, separated due to maturity of the CAT Bond in months
\[Issuedate\] binary variables, separated due to issue date of the CAT Bond in years
\[a, b, c_1, \ldots, \epsilon\] constants

The results of the stepwise multiple regression analysis are shown in Table 6. A high coefficient of determination of 0.860 has been achieved. The corresponding significant variable coefficients are shown in Table 7 and allow for an economic interpretation.

<table>
<thead>
<tr>
<th>Linear Model</th>
<th>(R)</th>
<th>(R^2)</th>
<th>adjusted (R^2)</th>
<th>Standard error of the estimator</th>
<th>Predictor variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Model</td>
<td>0.930</td>
<td>0.865</td>
<td>0.860</td>
<td>0.0146752</td>
<td>(constant), EL, Trigger_Parametric, Risk_Windstorm, Risk_Earthquake, Trigger_Indemnity, Rating_A</td>
</tr>
</tbody>
</table>

Table 6: Multiple Regression Analysis

As expected, the determined EL has a high significant impact on the observed one. A standardised loading factor of 0.884 is calculated for the EL. The newness of the asset class and the maturity term of CAT Bonds do not show any impact on the pricing mechanisms in our

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49 LANE/MAHUL (2008) received in their analysis a non standardised factor of 2.69 for the EL. It is similar to the non standardised factor of 2.46 which was received in the present analysis. The difference can be explained by the evaluation of more explaining factors in the present analysis.
study. Instead the high coefficient of determination can be explained in addition to the EL by the used trigger mechanism, the insured risk and the Rating.

Different trigger mechanisms refer to a different impact of basis risk and moral hazard on pricing. In literature it is stated, that a parametric trigger increases moral hazard significantly compared to indemnity trigger.\(^{50}\) Apart from that the trigger can be verified quickly. Thus, in particular investors are interested in CAT Bonds using a parametric trigger. Our results support this thesis, since the variable Trigger_Parametric has negative impact on pricing.

Although advantageous for investors, the use of parametric triggers contains increased basis risk for the sponsor. Recent market developments show that the indemnity trigger gain in importance due to reduced basis risk for the sponsor.\(^{51}\) This corresponds to the findings of our analysis, since the variable Trigger_Indemnity increases the price of a CAT Bond slightly. Market participants also tend to trust in A-rated Bonds specifically. The variable Rating_A has an increasing impact on the pricing.

<table>
<thead>
<tr>
<th>Linear Model</th>
<th>Model</th>
<th>non standardised coefficients</th>
<th>standardised coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dep. variable: (EL)</td>
<td>(constant)</td>
<td>0.043</td>
</tr>
<tr>
<td></td>
<td>(EL)</td>
<td>2.459</td>
<td>0.088</td>
</tr>
<tr>
<td></td>
<td>Trigger_Parametric</td>
<td>-0.016</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>Risk_Windstorm</td>
<td>-0.019</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>Risk_Earthquake</td>
<td>-0.012</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>Trigger_Indemnity</td>
<td>-0.009</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>Rating_A</td>
<td>-0.018</td>
<td>0.008</td>
</tr>
</tbody>
</table>

**Table 7: Coefficients of Regression Analysis**

Furthermore, the variables Risk_Windstorm and Risk_Earthquake show an increasing effect on prices. Thus we can assume that CAT Bonds insuring risks resulting from hurricanes or any combination of risks are imposed by the market with an additional risk load. Possibly this can be explained by the fact that the most severe losses to CAT Bonds were caused by hurricanes.\(^{52}\)

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\(^{50}\) See GUY CARPENTER (2007), p. 29.

\(^{51}\) See GUY CARPENTER (2008B), pp. 15.

\(^{52}\) See GUY CARPENTER (2006), p. 2.
4 Conclusion

Due to both an incomplete market for catastrophe risks and the lack of transparency on the CAT Bond market it is difficult to determine an accurate pricing model for CAT Bonds. Apart from that, the comparison of different CAT Bond pricing models remains a challenging question. This paper presented an overview of different pricing models for CAT Bonds. Especially premium calculation models which set the premiums for the transaction have been examined. These models have been used in order to carry out an empirical analysis with the goal of determining the most accurate model. As a measure for accuracy we used the fraction of variation which could be explained by the different models. The simple linear model described the data better than the generalised Wang transformation and the log-Regression. Thus, the linear model yielded in the most accurate model according to our method.

Further, an adjustment for cyclic, seasonal and business cyclic effects has been established with the objective of identifying their influence on different pricing models. Cyclic effects were recently discussed as a factor which influences prices for CAT Bonds. We verified this statement by using our method and an index provided by Lane Financial LLC. No model improvement has been identified, which could implicate either that the used index is not appropriate or that CAT Bond prices are not affected by cyclic effects. A further development of such indices with more data available could validate our findings. Moreover, also seasonal and business cyclic effects were not identified as influencing factors for CAT Bond prices. In particular, the latter supports the thesis of the independence of CAT Bond and capital markets, which is widely assumed in literature. A further analysis considering the effects of CAT Bond specific factors on the pricing of the Bonds using the linear model resulted in some interesting findings. Apart from the determined EL also some trigger mechanisms, specific risk classes and the Bond rating showed a significant impact on the pricing of CAT Bonds.
A Empirical Analysis

A.1 Generalised transformation

Using \( g_{\lambda}(u) = \Phi[a \cdot \Phi^{-1}(u) + \lambda] \) as the transformation function one receives with \( u = S_X(x) \) and \( g_{\lambda}(u) = S_X^*(x) \):

\[
S^*(x) = \Phi[a \cdot \Phi^{-1}(S(x) + \lambda)]
\]

\[\Leftrightarrow \Phi^{-1}(S^*(x)) = a \cdot \Phi^{-1}(S(x)) + \lambda \quad \forall a, \lambda \geq 0\]

A.2 Regression results

Assumptions for residuals

- Normal distribution:

The assumption of normal distributed residuals can be verified by examining the histograms. All residual statistics showed normal distributed residuals.

- Nonautocorrelation:

The Durbin-Watson Test is a statistical method for identifying autocorrelation of residuals. Possible values of the test statistic \( d \) are \( d \in [0,4] \). Values close to 2 allow the conclusion of nonautocorrelation. In practice it can be assumed, that \( 1.5 < d < 2.5 \) means no autocorrelation. Thus, our results barely lead to the conclusion of nonautocorrelated residuals.\(^{53}\)

<table>
<thead>
<tr>
<th>Model</th>
<th>Durbin-Watson Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Model</td>
<td>1.492</td>
</tr>
<tr>
<td>log-regression</td>
<td>1.592</td>
</tr>
<tr>
<td>Gen. Wang Transformation</td>
<td>1.53</td>
</tr>
</tbody>
</table>

### A.3 Regression results – Cyclic effects

#### I. Approach – multiple linear regression

<table>
<thead>
<tr>
<th>Model</th>
<th>non standardised coefficients</th>
<th>standardised coefficients</th>
<th>T</th>
<th>significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>standard error</td>
<td>Beta</td>
<td></td>
</tr>
<tr>
<td>Linear Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(constant)</td>
<td>0.035</td>
<td>0.009</td>
<td>3.789</td>
<td>0.000</td>
</tr>
<tr>
<td>PFL</td>
<td>2.302</td>
<td>0.086</td>
<td>0.874</td>
<td>26.729</td>
</tr>
<tr>
<td>CI_i</td>
<td>0.005</td>
<td>0.008</td>
<td>0.020</td>
<td>0.620</td>
</tr>
<tr>
<td>log-Regression</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(constant)</td>
<td>-0.748</td>
<td>0.157</td>
<td>-4.772</td>
<td>0.000</td>
</tr>
<tr>
<td>ln(PFL)</td>
<td>0.439</td>
<td>0.021</td>
<td>0.813</td>
<td>20.702</td>
</tr>
<tr>
<td>CI_i</td>
<td>0.066</td>
<td>0.112</td>
<td>0.023</td>
<td>0.594</td>
</tr>
<tr>
<td>Gen. Wang Transformation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(constant)</td>
<td>-0.028</td>
<td>0.092</td>
<td>-0.305</td>
<td>0.761</td>
</tr>
<tr>
<td>PFL^-1(CI_i)</td>
<td>0.647</td>
<td>0.029</td>
<td>0.833</td>
<td>22.304</td>
</tr>
<tr>
<td>CI_i</td>
<td>0.033</td>
<td>0.058</td>
<td>0.021</td>
<td>0.569</td>
</tr>
</tbody>
</table>

Residuals:

<table>
<thead>
<tr>
<th>Model</th>
<th>Durbin-Watson Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Model</td>
<td>1.507</td>
</tr>
<tr>
<td>log-regression</td>
<td>1.605</td>
</tr>
<tr>
<td>Gen. Wang Transformation</td>
<td>1.546</td>
</tr>
</tbody>
</table>

#### II. Approach – adjusted probabilities

<table>
<thead>
<tr>
<th>Model</th>
<th>$R$</th>
<th>$R^2$</th>
<th>adjusted $R^2$</th>
<th>Standard error of the estimator</th>
<th>Predictor variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Model</td>
<td>0.817</td>
<td>0.667</td>
<td>0.665</td>
<td>0.03191687</td>
<td>(constant), $PFL$</td>
</tr>
<tr>
<td>Dep. Variable: $PFL/CI_i$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log-regression</td>
<td>0.663</td>
<td>0.439</td>
<td>0.437</td>
<td>0.592626428</td>
<td>(constant), ln($PFL$)</td>
</tr>
<tr>
<td>Dep. Variable: ln($PFL$)/CI_i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gen. Wang Transformation</td>
<td>0.683</td>
<td>0.466</td>
<td>0.464</td>
<td>0.3169002</td>
<td>(constant), $\Phi^{-1}(PFL)$</td>
</tr>
<tr>
<td>Dep. Variable: $\Phi^{-1}(PFL)/CI_i$</td>
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</table>
### A.4 Regression results – Seasonal effects

<table>
<thead>
<tr>
<th>Model</th>
<th>( R )</th>
<th>( R^2 )</th>
<th>adjusted ( R^2 )</th>
<th>Standard error of the estimator</th>
<th>Predictor variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Model</td>
<td>0.876</td>
<td>0.768</td>
<td>0.766</td>
<td>0.028</td>
<td>(constant), ( PFL ), ( SI _S )</td>
</tr>
<tr>
<td>log-regression</td>
<td>0.815</td>
<td>0.664</td>
<td>0.660</td>
<td>0.366</td>
<td>(constant), ( \ln(PFL) ), ( SI _S )</td>
</tr>
<tr>
<td>Gen. Wang Transformation</td>
<td>0.834</td>
<td>0.696</td>
<td>0.694</td>
<td>0.188</td>
<td>(constant), ( \Phi^{-1}(PFL) ), ( SI _S )</td>
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</table>

Assumptions:

<table>
<thead>
<tr>
<th>Model</th>
<th>Durbin-Watson Test</th>
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</thead>
<tbody>
<tr>
<td>Linear Model</td>
<td>1.502</td>
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<tr>
<td>log-regression</td>
<td>1.592</td>
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<tr>
<td>Gen. Wang Transformation</td>
<td>1.540</td>
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</table>
### A.5 Regression results – business cyclic effects

<table>
<thead>
<tr>
<th>Model</th>
<th>( R )</th>
<th>( R^2 )</th>
<th>adjusted ( R^2 )</th>
<th>Standard error of the estimator</th>
<th>Predictor variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Model</td>
<td>0.876</td>
<td>0.767</td>
<td>0.765</td>
<td>0.0277</td>
<td>(constant), ( PFL ), GDP</td>
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<tr>
<td>log-regression</td>
<td>0.815</td>
<td>0.665</td>
<td>0.662</td>
<td>0.366</td>
<td>(constant), ( \ln(PFL) ), GDP</td>
</tr>
<tr>
<td>Gen. Wang Transformation</td>
<td>0.835</td>
<td>0.697</td>
<td>0.695</td>
<td>0.188</td>
<td>(constant), ( \Phi^{-1}(PFL) ), GDP</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>Non standardised coefficients</th>
<th>Standardised coefficients</th>
<th>( T )</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Model</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>dep. variable: ( PFL )</td>
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<tr>
<td>( PFL )</td>
<td>2.307</td>
<td>0.086</td>
<td>0.876</td>
<td>26.879</td>
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<tr>
<td>GDP</td>
<td>7.037E-5</td>
<td>0.001</td>
<td>0.002</td>
<td>0.072</td>
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<tr>
<td>(constant)</td>
<td>0.041</td>
<td>0.004</td>
<td>10.834</td>
<td>0.000</td>
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<tr>
<td>dep. variable: ( GDP )</td>
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</tr>
<tr>
<td>(constant)</td>
<td>-0.698</td>
<td>0.097</td>
<td>-7.237</td>
<td>0.000</td>
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<tr>
<td>log-regression</td>
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<td></td>
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<tr>
<td>dep. variable: ( \ln(PFL) )</td>
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<tr>
<td>( \ln(PFL) )</td>
<td>0.442</td>
<td>0.021</td>
<td>0.819</td>
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<tr>
<td>GDP</td>
<td>0.013</td>
<td>0.013</td>
<td>0.041</td>
<td>1.037</td>
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<tr>
<td>(constant)</td>
<td>-0.698</td>
<td>0.097</td>
<td>-7.237</td>
<td>0.000</td>
</tr>
<tr>
<td>Gen. Wang Transformation</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dep. variable: ( \Phi^{-1}(PFL) )</td>
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</tr>
<tr>
<td>( \Phi^{-1}(PFL) )</td>
<td>0.651</td>
<td>0.029</td>
<td>0.838</td>
<td>22.442</td>
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<tr>
<td>GDP</td>
<td>0.007</td>
<td>0.007</td>
<td>0.039</td>
<td>1.050</td>
</tr>
</tbody>
</table>


