Review of Exponents

You should convince/remind yourself of the following:

$$x^2 = x \cdot x$$

$$x^a \cdot x^b = x^{a+b} \quad (e.g.\ 2^2 \cdot 2^3 = 2^5 = 32 = 4 \cdot 8)$$

$$\left(x^a\right)^b = x^{ab} \quad (e.g.\ \left(2^2\right)^3 = 2^6 = 64)$$

$$x^{-a} = \frac{1}{x^a} \quad \left(e.g.\ 3^{-2} = \frac{1}{3^2} = \frac{1}{9}\right)$$

$$x^{\frac{1}{2}} = \sqrt{x} \quad \left(e.g.\ 9^{\frac{1}{2}} = \sqrt{9} = 3\right)$$

Natural Logarithms

Definition of a Logarithm:

$y = \log_b x \text{ if and only if } x = b^y \left(b > 0, b \neq 1\right)$

The logarithm of a number x to a base b ($b > 0$, $b \neq 1$) is the exponent to which b must be raised to equal x.

We're going to be dealing with a particular base for our logarithms. This base will be the number **e**.

Where:

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \approx 2.718281828459045$$

e is like in that it represents a particular and very useful number.

Note that sometimes e is expressed as "exp" for "exponent" so that you should read **exp(rt)** as $\mathbf{e^{rt}}$**.**

Logarithms with this base are called *natural logarithms* and are often written as *ln*.

All of the properties of logs apply to natural logarithms.

You will see in Lecture 2 why e is a useful number in finance.