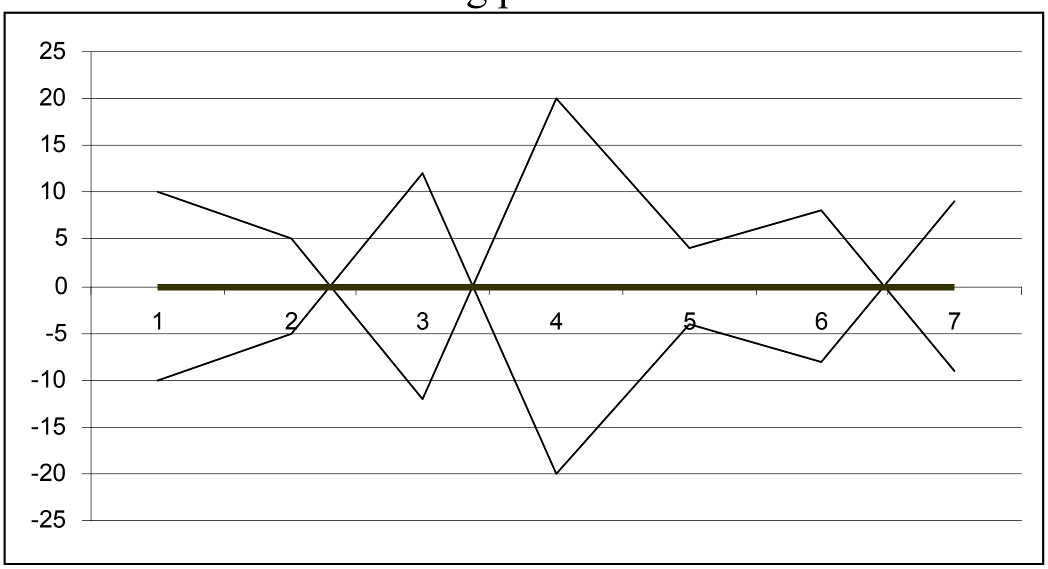
THE RISK OF A PORTFOLIO OF SECURITIES

What is a portfolio?

Portfolio: A collection of assets such as stocks, bond, real estate, etc.

The return to a portfolio of assets is an average return of all of the assets, weighted by the relative amount invested in each asset. However, the risk of a portfolio of assets is fundamentally different from the average risk of all assets in the portfolio.

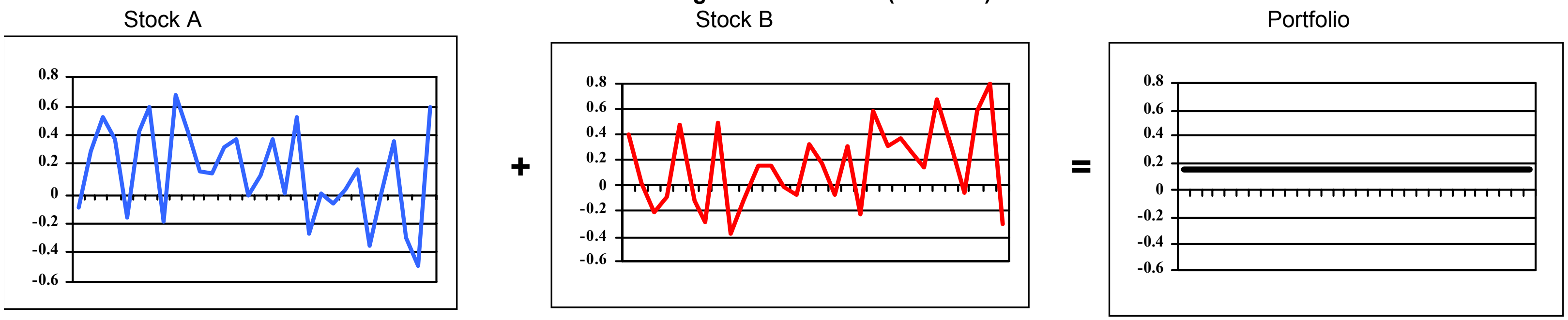
The process of ***diversification*** will reduce the risk of assets when held in portfolios. Think of the following picture of two asset's returns:



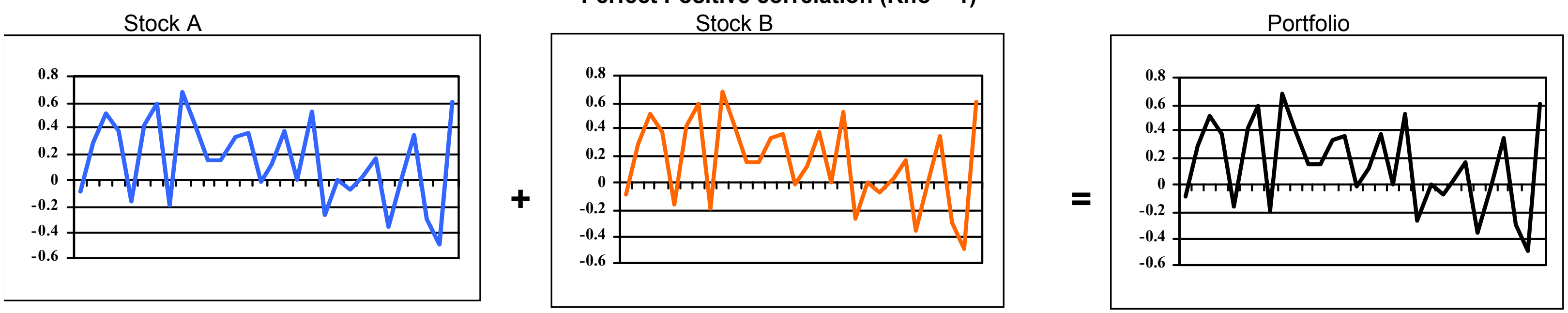
The riskiness of a portfolio depends not only on the riskiness of each asset in the portfolio, but also on the relation between the returns from the two assets.

We talk about the ***correlation*** between two assets. Correlation measures the degree to which when one asset goes up the other asset also goes up.

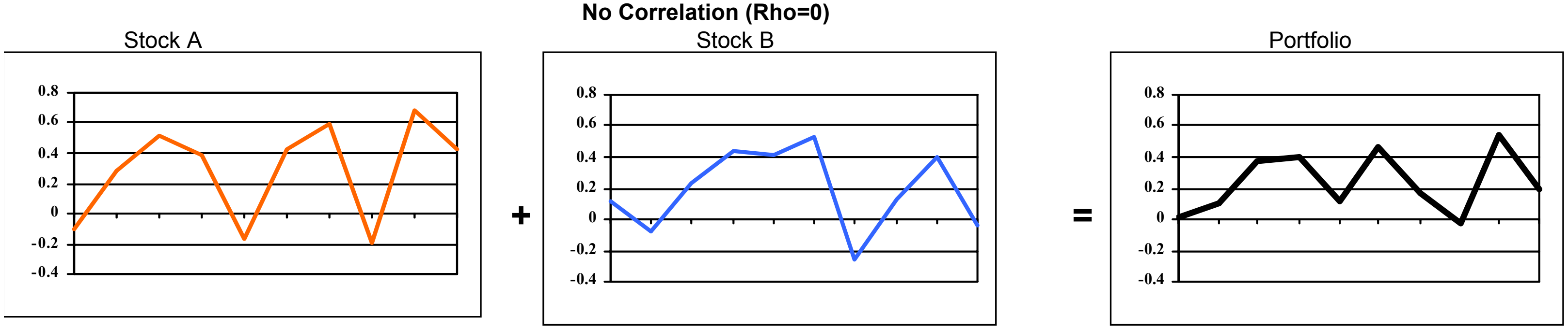
Perfect Negative correlation (Rho = -1)



Perfect Positive correlation (Rho = 1)



No Correlation (Rho=0)



Investors care about the future **rate of return** and **risk** of their portfolios

It is useful to characterize (measure) these two attributes as follows:

Return: expected, or average, return

Risk: dispersion of return, measured by variance or standard deviation

An important question: what does an individual asset contribute to portfolio expected return and risk (dispersion)?

Attribute	Contribution to Portfolio
Expected Return	The asset's own expected return multiplied by its importance (weight) in the portfolio
Risk (variability)	The asset's own variability, but more importantly, the asset's covariability (correlation) with other assets in the portfolio

From a portfolio perspective, an individual asset's risk is **what it contributes to portfolio variability or risk**

An asset's own variability (standard deviation) can be partitioned into two components:

Non-diversifiable portion

Diversifiable portion

The <u>**non-diversifiable portion**</u> is the risk of the asset that the investor bears when holding the asset in his/her portfolio

Now, let's do a bit of math and then continue discussing the intuition.

The Mathematical formula for Portfolio Risk

Two Assets: A & B

w_A and w_B are the fractions of total funds invested in each asset.

σ_A and σ_B are the standard deviations of returns for each asset.

ρ_{AB} is the correlation between the two assets' returns. It must lie between –1 and +1.

$$\rho_{AB} = \frac{Cov(R_A, R_B)}{\sigma_A \cdot \sigma_B}$$

$$Cov(R_A, R_B) = \sum_{i=1}^{n} p_i (R_A - \overline{R}_A)(R_B - \overline{R}_B)$$

With these definitions we can write down the variance and standard deviation of a portfolio of 2 assets.

A Two-Asset Portfolio:

Variance:

$$\sigma_P^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B Cov(R_A, R_B) \qquad (1)$$

Now, using the definition of correlation :

$$Cov(R_A, R_B) = \sigma_A \cdot \sigma_B \cdot \rho_{AB}$$

$$\sigma_P^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \cdot \sigma_A \cdot \sigma_B \cdot \rho_{AB} \qquad (2)$$

$$\sigma_P = \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \cdot \sigma_A \cdot \sigma_B \cdot \rho_{AB}}$$

ρ_{AB} varies between -1 and 1.

Using either equation (1) or (2) will give us the same answer with just slightly different inputs.

	Stock A	**Stock B**
Stock A		
Stock B		

Example:

Risk of a Portfolio is NOT just the average risk of the securities

Asset	E[R]	σ	Weight
A	12%	20%	2/3
B	15%	40%	1/3

Weighted average of the expected returns is 13%.

$E[R_P] = w_A E[R_A] + w_B E[R_B]$

$=$

$=$

Weighted average of standard deviations is 26.67%

E[σ] =

=

But the standard deviation of the portfolio is going to depend upon the correlation (ρ) between asset A and asset B. Why?

Say ρ = 0.5

$$\sigma_P = \sqrt{w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \cdot \sigma_A \cdot \sigma_B \cdot \rho_{AB}}$$

$$\sigma_P = \sqrt{\left(\frac{2}{3}\right)^2 (0.2)^2 + \left(\frac{1}{3}\right)^2 (0.4)^2 + 2\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) \cdot 0.2 \cdot 0.4 \cdot 0.5} = 23.09\%$$

Say $\rho = 0$

$$\sigma_P = \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \cdot \rho_{AB} \cdot \sigma_A \cdot \sigma_B}$$

$$\sigma_P =$$

Say $\rho = -0.5$

$$\sigma_P = \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \cdot \rho_{AB} \cdot \sigma_A \cdot \sigma_B}$$

$$\sigma_P =$$

What does this say about choosing a portfolio of securities?

- It says that if you select stocks that are not perfectly positively correlated, you can substantially reduce the risk of your portfolio.
- This rule basically says do not put all your eggs in one basket.
- You need to be concerned about the correlation between stocks in your portfolio because that will determine the non-diversifiable portion of your stocks' risks.

Recall, before the math we said that any asset's risk can be partitioned into non-diversifiable and diversifiable components. This partition of asset variability into non-diversifiable and diversifiable components is portfolio-specific; that is, the partition is in general different for different portfolios

One way around this limitation of a portfolio-specific risk measure is to define a sensible, market-wide "benchmark" portfolio to which all investors can relate.

The 'Market Portfolio', consisting (in principle) of all risky securities, is such a benchmark.

- it is not investor-specific
- it represents the limit of diversification

To summarize, an asset's components of risk can be defined as:

Non-diversifiable	**Diversifiable**

These components are also known as –

market risk	unique risk
systematic risk	unsystematic risk
general risk	specific risk
	residual risk

The importance of the distinction between non-diversifiable risk and diversifiable risk is that **non-diversifiable risk** is likely to be **compensated risk** (via higher expected return), whereas **diversifiable risk** is **not** likely to be compensated (since it can be eliminated via diversification)

SYSTEMATIC AND UNSYSTEMATIC RISK

1. Unsystematic risk (diversifiable or company specific risk).

This is risk that gets washed away by opposite/uncorrelated events occurring to companies in the portfolio which you hold.

Examples

- Fire at company headquarters
-
-
-

2. Systematic Risk (Nondiversifiable risk or Market risk):

The part of risk that can not be eliminated through diversification. Events which "systematically" affect all stocks.

- Interest rates
-
-

Which risks can holding a portfolio help you avoid?

Which risks should you be rewarded for bearing?

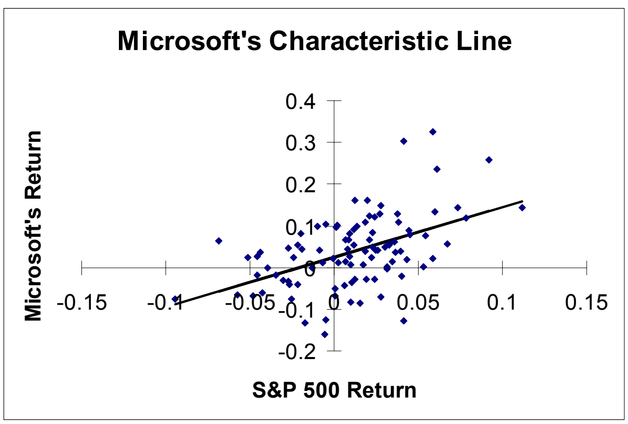
MEASURING SYSTEMATIC RISK:

What is the largest portfolio we can invest in?

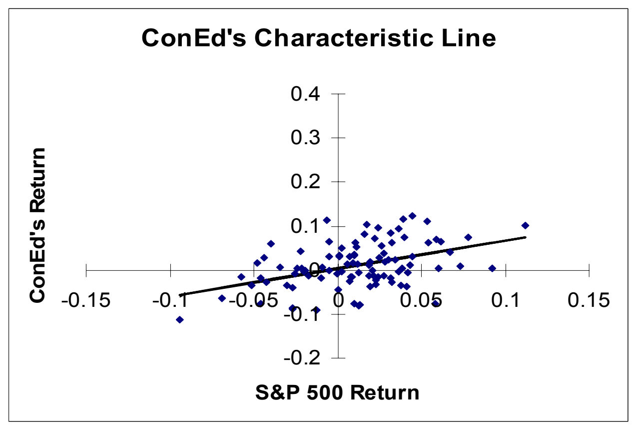
This portfolio by definition has no unsystematic risk.

Therefore, the covariance of a stock with the market portfolio provides a measure of the risk which can not be diversified away.

The ***characteristic line*** shows the relation between a security's returns and the market's return. The slope of the characteristic line is called the beta (β). The β measures the stock's systematic risk.



$\beta = 1.2$



$\beta = 0.67$

Beta

An asset's beta coefficient is a measure of its **non-diversifiable** risk

$$\text{Beta} = \frac{\text{asset's standard deviation} \times \text{asset's correlation with Market Portfolio}}{\text{standard deviation of Market Portfolio}}$$

Beta is a measure of the asset's non-diversifiable risk **relative to the risk of the Market Portfolio**

Beta of the Market Portfolio = 1.0

A Fundamental Premise

The collective behavior of security market participants is characterized by **risk aversion**

therefore, investments with high risk must be accompanied by high expected return

A simple model:

Investors demand a base level rate of return even for a risk-free investment

$r_{rf} \equiv$ the rate of return on a risk-free asset

Investors demand a **risk premium** (i.e., additional *expected* but uncertain rate of return) to compensate for the risk of the investment; so

required (minimum acceptable) rate of return on investment = r_{rf} + risk premium

Putting the Pieces Together

Risk Premium for a risky asset	=	Risk premium per unit of non-diversifiable risk	X	Number of units of non-diversifiable risk
	=	$(r_m - r_{rf})$	X	**Beta of asset**
Required (minimum acceptable) rate of return on a risky asset	=	Risk-free rate	+	Risk premium
r_{asset}	=	r_{rf}	+	$(r_m - r_f) \times \beta_{asset}$

The equation above is the **CAPM** (capital asset pricing model)

Historical evidence of the risk premium on a broad portfolio of common stocks (Ibbotson - Sinquefield data from 1926 - 2000)

$$r_m - r_{rf} \approx 9\%$$

where

$r_m \equiv$ average return on the Market Portfolio

$r_{rf} \equiv$ average return on the risk-free asset

Note that since the beta of the risk-free asset is zero, and the beta of the Market Portfolio is 1.0,

9% is the approximate historical risk premium <u>per unit</u> of non-diversifiable risk

$$\frac{\mathbf{r_m - r_f}}{\mathbf{1.0 - 0}} \approx \mathbf{9\%}$$

An asset's β is used to calculate its expected return using the CAPM:

$$E[r_i] = r_{rf} + \beta_i (E[r_m] - r_{rf})$$

$E[r_i]$ is the expected return on asset i

r_{rf} is the risk - free rate

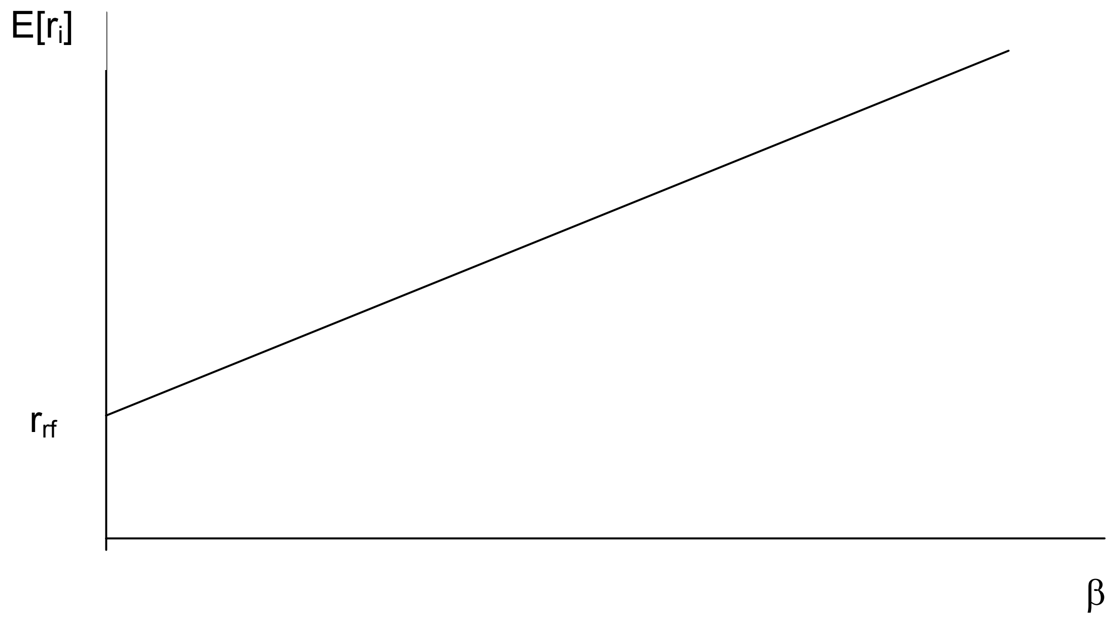
β_i is asset i's units of systematic risk

$E[r_m]$ is the expected return on the market portfolio

$(E[r_m] - r_{rf})$ is the expected market risk premium

The Security Market Line (SML): $E[r_i] = r_{rf} + \beta_i(r_m - r_{rf})$

Provides the level of expected return for various values of β.



Suppose the risk-free rate is 5%, the return on the market is 15% and you have calculated the β of your portfolio to be 1.8. What is the expected return on your portfolio?

$$E[r_P] = r_{rf} + \beta_P(r_m - r_{rf})$$

$$E[r_P] =$$

$$E[r_P] =$$